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Critical sizes of ground and underground horizontal cylindrical tanks

K. Magnucki [a,b,*], P. Stasiewicz [a]

[a] *Institute of Applied Mechanics, Poznań University of Technology, ul. Piotrowo 3, 60-965 Poznań, Poland*
[b] *Institute of Rail Vehicles 'TABOR', ul. Warszawska 181, 61-055 Poznań, Poland*

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Abstract

The paper deals with ground and underground horizontal cylindrical tanks supported at both ends. The ground tanks are loaded with internal hydrostatic pressure and small negative pressure. On the other hand, the underground tanks are located in water containing soil and loaded with external hydrostatic pressure. Critical states of both structures are determined based on solving the equation of stability of cylindrical shell. The problem so defined has been converted to calculation of critical thicknesses of walls for the family of circular cylindrical tanks of different capacities. Critical sizes of the structures have been determined as functions of dimensionless critical thickness and dimensionless tank length.

Keywords: Stability of shells; Thin-walled cylindrical shell; Horizontal tanks; Critical state

1. Introduction

In order to design horizontal cylindrical tanks the strength and stability conditions must be taken into account. Huang et al. [1] analytically and numerically determined critical load of a horizontal circular cylindrical shell subject to three forces distributed on small surfaces. Such a load causes a pure bending and occurs, first of all, in industrial pipelines. Guarracino and Fraldi [2] analytically determined deformation of circular cross section of a pipe subject to pure bending. Chan et al. [3, 4] presented

* Corresponding author.
E-mail address: krzysztof.magnucki@put.poznan.pl (K. Magnucki).

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Nomenclature

a_{ij}	matrix coefficients
b	convexity of head of tank
n	natural number
t	thickness of the cylindrical shell
h_0	depth of foundation
w_i	parameters of deflection function
α_k	pressure parameters
γ_{m1}, γ_{m2}	specific weight of liquid
L	length of the cylindrical shell
R	radius of the cylindrical shell
V_0	capacity of the tank

research results related to stability of horizontal shells pivoted at both ends and filled with a fluid. They proposed expressions of admissible stresses using known analytical solutions for axial compression or pure bending of cylindrical shell. They compared the results of their experiment with appropriate values determined based on British and European Standards. Kacperski [5] carried out an experimental research of a real horizontal cylindrical vessel filled with water and pivoted at both ends. Increasing its length by welding further cylindrical segments, maintaining their constant diameter and wall thickness, he caused a loss of stability of the structure. The wall buckled in middle part of the tank. Saal [6] carried out experimental laboratory investigation of buckling process of horizontal thin cylindrical shells filled with water. He determined a proportion between the dimensions of cylindrical shells for various levels of water, for which the structure lost its stability. Magnucki and Stasiewicz [7, 8] by means of FEM analytically and numerically determined critical state of horizontal cylindrical shell filled with liquid and supported at both ends. Stasiewicz [9] presented further steps of stability research of such a shell. Magnucki [10] provided optimal dimensions of horizontal cylindrical tanks with convex heads, filled with water, taking into account the strength and stability conditions. Ziółko [11] provided detailed principles for designing of the tanks of various shapes, considering their real manufacturing conditions. The conditions determining shaping of the structures result chiefly from their purposes and all the factors affecting security of their use should be taken into consideration.

2. Stability of horizontal cylindrical tanks

Ground and underground horizontal tanks supported at both ends have been analyzed (Fig. 1). The ground tanks filled with a liquid are loaded with internal hydro-

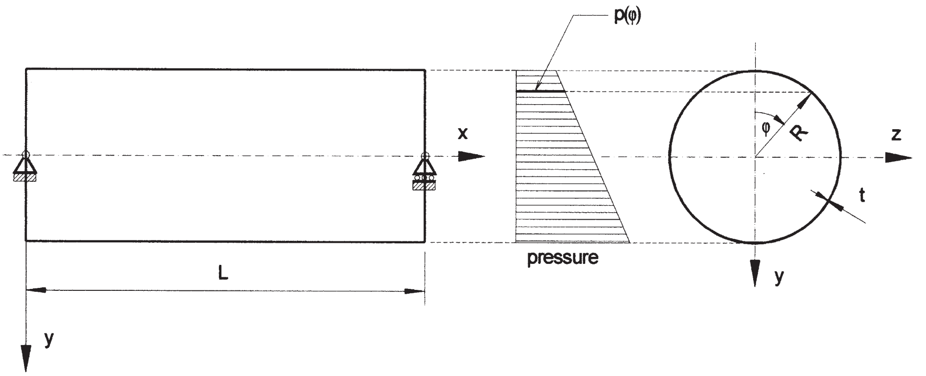

Fig. 1. Ground and underground horizontal tanks.

static pressure and small negative pressure (external pressure) p_{01}, therefore total pressure

$$p(\varphi) = \gamma_{m1}R(1-\cos\varphi)-p_{01} = \gamma_m R(1-2\alpha_1-\cos\varphi), \quad (1)$$

where: R is the radius of cylindrical shell; γ_{m1} is the specific weight of liquid; $\alpha_1 = \frac{p_{01}}{2\gamma_{m1}R}$ is the pressure parameter. The underground tanks are empty and located in water containing soil at the depth h_0, therefore their sole load is an external hydrostatic pressure

$$p(\varphi) = -\gamma_{m2}R(1-\cos\varphi)-\gamma_{m2}h_0 = -\gamma_{m2}R(1-2\alpha_2-\cos\varphi), \quad (2)$$

where: γ_{m2} is the specific weight of water; $\alpha_2 = -\frac{h_0}{2R}$ is the pressure parameter.

Forces of the pre-buckling state [10], determined based on linear equations of equilibrium of cylindrical shell, are equal to:

$$N^0_{xx} = \frac{4}{\pi^3}\gamma_k L^2\cos\varphi\cdot\cos\frac{\pi x}{L} + \alpha_k\gamma_k R^2,\ N^0_{x\varphi} = \frac{4}{\pi^2}\gamma_k LR\sin\varphi\cdot\sin\frac{\pi x}{L},\ N^0_{xx} = \frac{4}{\pi}\gamma_k R^2(\cos\varphi-1)\cos\frac{\pi x}{L} + 2\alpha_k\gamma_k R^2,\ k = 1,2 \quad (3)$$

where: $\gamma_1 = \gamma_{m1}$, $\gamma_2 = -\gamma_{m2}$.

Donnell's equation of stability of cylindrical shells [12] (Fig. 2) in x, φ coordinates may be written as follows

$$D\nabla^8 w + \frac{Et\partial^4 w}{R^2\partial x^4} + \nabla^4\left(N^0_{xx}\frac{\partial^2 w}{\partial x^2} + 2N^0_{x\varphi}\frac{\partial^2 w}{R\partial x\partial\varphi} + N^0_{\varphi\varphi}\frac{\partial^2 w}{R^2\partial\varphi^2}\right) = \nabla^4 p, \quad (4)$$

where: $w = w(x,\varphi)$ is the deflection of the shell; $D = \frac{Et^3}{12(1-\nu^2)}$ is the bending stiff-

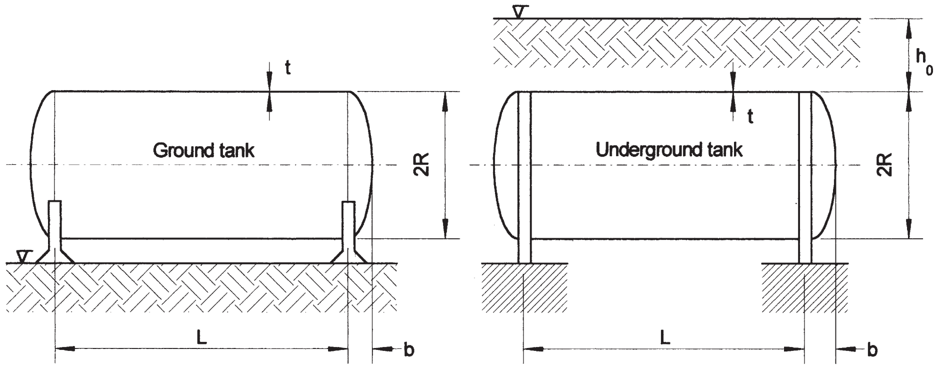

Fig. 2. Cylindrical shell of horizontal tanks.

ness; E i ν is Young's modulus and Poisson ratio; t is the thickness of the shell, $\nabla^8 = (\nabla^2)^4, \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{R^2 \partial\varphi^2}$ is the Laplace operator.

The equation of stability for the analyzed load case may be solved only approximately. It is useful to make use of Galerkin's method that required assuming a deflection function. It should be noticed that selection of appropriate deflection function is of great importance for the solution. Experimental research carried out by Kacperski [5] and Saal [6] are very helpful in this case. Taking into account the shape of buckled structure the deflection of the shell has been assumed in the form of the following function

$$w(x,\varphi) = t\left(w_2 + 2\sum_{i=3}^{5} w_i \cos(i-2)\varphi\right)\cos n\varphi \cdot \cos\frac{\pi x}{L}, \tag{5}$$

where: w_i is the free parameters of deflection function ($i = 2,3,4,5$); n is the natural number ($n = 2,3,4,...$).

For this purpose it is assumed that boundaries of the cylindrical shell are pivoted (Fig. 2), i.e. the deflection and bending moment are equal to zero. Therefore, the function (5) meets such an assumption. Thus, Galerkin's ortogonalization condition is of the form

$$\int_{-L/2}^{L/2}\int_{0}^{2\pi}\left[D\nabla^8 w + \frac{Et\partial^4 w}{R^2\partial x^4} + \nabla^4\left(N_{xx}^0\frac{\partial^2 w}{\partial x^2} + 2N_{x\varphi}^0\frac{\partial^2 w}{R\partial x\partial\varphi} + N_{\varphi\varphi}^0\frac{\partial^2 w}{R^2\partial\varphi^2} - p(\varphi)\right)\right]\cdot f_i(\varphi)\cos\frac{\pi x}{L}d\varphi dx = 0$$

where: $f_i(\varphi) = \cos(i-2)\varphi \cdot \cos n\varphi$, $i = 2,3,4,5$.

Hence, integration provides four homogeneous linear algebraic equations

$$\begin{bmatrix} a_{22} & a_{23} & 0 & 0 \\ -a_{32} & a_{33} & -a_{34} & 0 \\ 0 & -a_{43} & a_{44} & -a_{45} \\ 0 & 0 & -a_{54} & a_{55} \end{bmatrix} \bullet \begin{Bmatrix} w_2 \\ w_3 \\ w_4 \\ w_5 \end{Bmatrix} = \vec{0}, \tag{6}$$

where:

$$a_{23} = \frac{8}{3\pi^2}\left[3\frac{\lambda^4}{\pi^4}n^6 + (n^2 + 4)(4 + C_n^2)^2\right]$$

$$a_{32} = \frac{2}{3\pi^2}\left[3\frac{\lambda^4}{\pi^4}((n-1)^6 + (n+1)^6) + (n+1)^2(4 + C_{n-1})^2 + (n-1)^2(4 + C_{n+1})^2\right]$$

$$a_{33} = \frac{Et}{\gamma_k R^2}a_{331} + \frac{32}{3\pi^2}a_{332} - \frac{\pi^2}{\lambda^2}\alpha_k a_{333},\ a_{331} = 1 + \frac{1}{2}C_0[(1 + C_{n-1})^4 + (1 + C_{n+1})^4]$$

$$a_{332} = \frac{1}{2}\{(n-1)^2[3 + (1 + C_{n-1})^2] + (n+1)^2[3 + (1 + C_{n+1})^2]\}$$

$$a_{333} = \frac{1}{2}[(1 + 2C_{n-1})(1 + C_{n-1})^2 + (1 + 2C_{n+1})(1 + C_{n+1})^2]$$

$$a_{34} = \frac{2}{3\pi^2}\left[3\frac{\lambda^4}{\pi^4}((n-1)^6 + (n+1)^6) + (n-3)^2(4 + C_{n-1})^2 + (n+3)^2(4 + C_{n+1})^2\right]$$

$$\lambda = \frac{L}{R},\ C_0 = \frac{\pi^4}{12(1-\nu^2)}\frac{t^2}{R^2\lambda^4},\ C_{n\pm r} = (n \pm r)^2\frac{\lambda^2}{\pi^2},\ r = 0, 1, 2, 3.$$

Another coefficients a_{ij} are similar, therefore not presented here.

A condition for existence of the solution is zero value of determinant of the system of Eq. (6), i.e.

$$\det\begin{bmatrix} a_{22} & a_{23} & 0 & 0 \\ -a_{32} & a_{33} & -a_{34} & 0 \\ 0 & -a_{43} & a_{44} & -a_{45} \\ 0 & 0 & -a_{54} & a_{55} \end{bmatrix} = 0. \tag{7}$$

Coefficients a_{ij} of the equation include material constants (E, ν), specific weight of the liquid γ_k, pressure parameter, α_k, and basic dimensions of the cylindrical shell. Therefore, taking into account the load conditions, solution of the Eq. (7) expresses a relationship between three basic dimensions of the shell (thickness, t; radius, R; length, L) determining critical state of the structure.

3. Critical sizes of horizontal cylindrical tanks

A horizontal cylindrical tank (Fig. 1) is composed of a cylindrical shell, two heads and two supports. The heads are usually convex. For the analysis a tank with classic ellipsoidal heads has been assumed, of relative height $\beta = b/R = 0.5$. Capacity of such a tank is a sum of capacity of two heads and that of the cylindrical shell

$$V_0 = 2V_d + V_w, \tag{8}$$

where: $V_d = \frac{\pi}{3}R^3$ is the capacity of ellipsoidal head ($b = 0.5R$); $V_w = \pi R^2 L$ is the capacity of cylindrical shell. Hence, the radius of the tank, i.e. cylindrical shell

$$R = \sqrt[3]{\frac{3V_0}{\pi(2 + 3\lambda)}}. \tag{9}$$

On substituting the expression into the coefficients a_{ij} of stiffness matrix of the shell, the condition (7) enables numerical determining of critical value of relative wall thickness $(t/R)_{CR}$ of the tank, for its fixed volume V_0 and various relative length values $\lambda = L/R$.

4. Numerical analysis

The research is made on a family of circular cylindrical tanks of capacities $V_0 = 100$, 200, and 300 m^3. Material constants of the steel: Young's modulus $E = 2.1 \cdot 10^5$ MPa Poisson ratio $\nu = 0.3$. Ground tanks are filled with water of specific weight $\gamma_m = 9.81$ kN/m^3 and loaded with additional external pressure $p_0 = 0.01$ MPa or $p_0 = 0.1$ MPa. The underground tanks are empty and located in water containing soil ($\gamma_m = 9.81$ kN/m^3) at two depths $h_0 = 1$ m or $h_0 = 10$ m. Supports of the tanks are of anchor type and prevent them from floating up to the ground surface.

Based on the condition (7) relative critical thickness values $(t/R)_{KR}$ of the walls of examined tanks have been calculated. Results of the analysis for ground tanks are presented in Figs. 3 and 4, while those for the underground ones are shown in Figs. 5 and 6. Bottom curves are related to the volume $V_0 = 100$ m^3, while the upper ones are related to the volume $V_0 = 300$ m^3. The distance between these curves depends on the values of additional negative pressure or on the depth of tank foundation (pressure parameters α_1, α_2). For the ground tanks loaded with greater negative pressure $p_0 = 0.1$ MPa the curves almost coincide each other (Fig. 4), while for underground tanks founded at small depth $h_0 = 1$ m the curves are noticeably separated (Fig. 5). The region below the curves corresponds to unstable state, while over them corresponds to a stable one. The curves located at the border of both regions describe critical state of the structure. For purpose of practical calculations the use of envelope of the curves is more convenient. The envelopes for consecutive families of the curves have been approximated by a function in the form

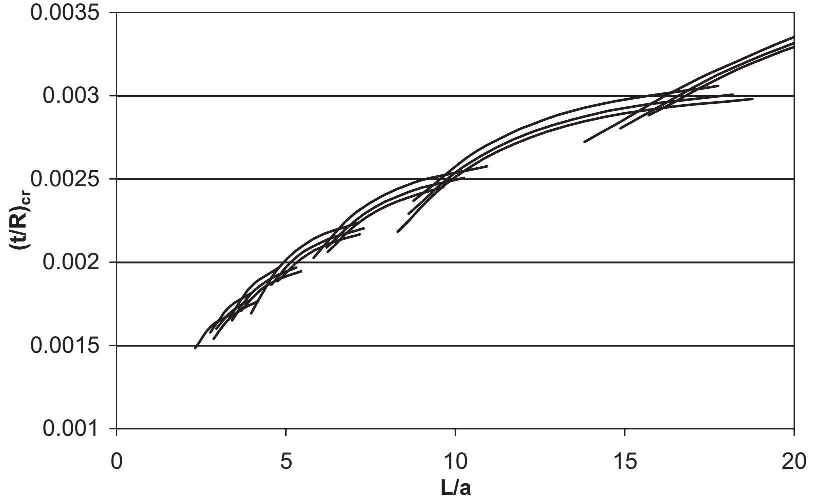

Fig. 3. Relative critical thickness of ground tanks (p_0 = 0.01 MPa).

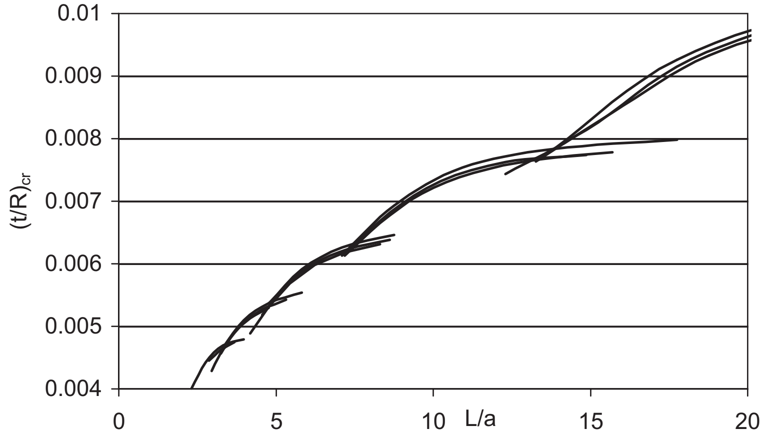

Fig. 4. Relative critical thickness of ground tanks (p_0 = 0.1 MPa).

$$\left(\frac{t}{R}\right)_{CR} = \beta_1\left(\frac{L}{R}\right)^{\beta_2}, \tag{10}$$

where: β_1, β_2 are the parameters the values of which for the ground tanks are shown in Table 1, while for the underground ones are shown in Table 2. Due to proximity of the curves for underground tanks and negative pressure $p_0 = 0.01$ MPa (Fig. 3) only two volumes $V_0 = 100\ \mathrm{m}^3$ (the bottom curves) and $V_0 = 300\ \mathrm{m}^3$ (the upper curves) have been introduced. The difference of critical wall thicknesses for these tanks is less than 4%. On the other hand, for the ground tanks and negative pressure

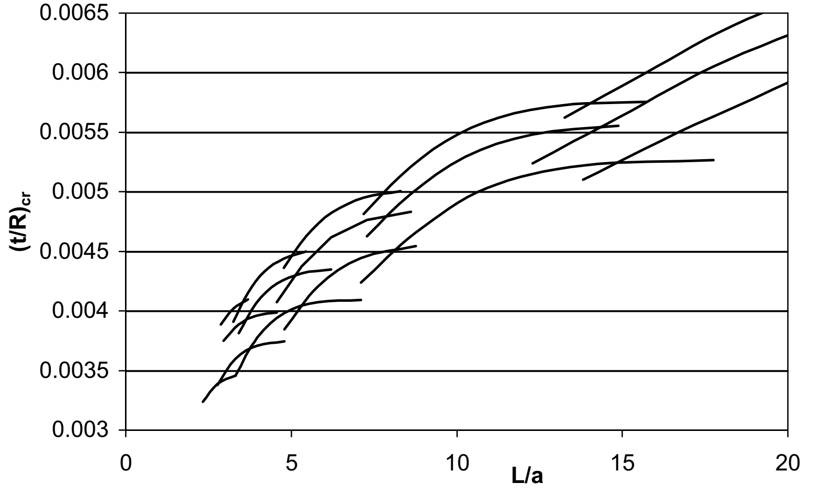

Fig. 5. Relative critical thickness of underground tanks (h_0 = 1 m).

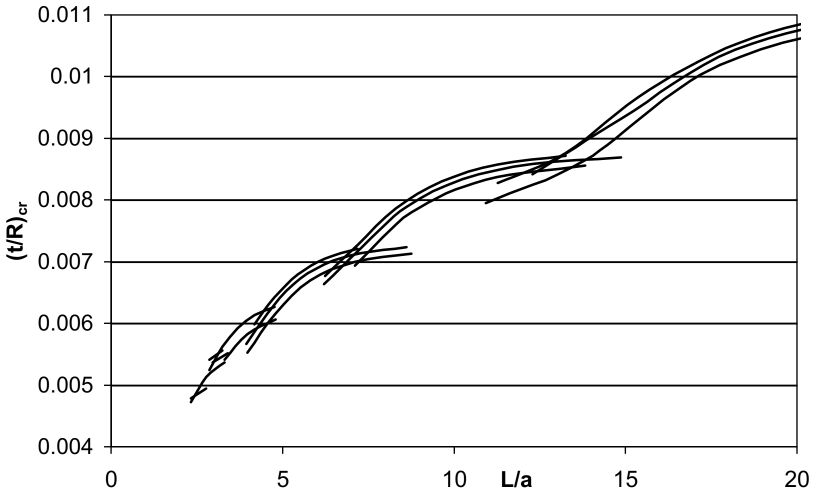

Fig. 6. Relative critical thickness of underground tanks (h_0 = 10 m).

p_0 = 0.1 MPa (Fig. 4) only one envelope for the volume V_0 = 300 m^3 is left, as for the other volumes the curves are very similar.

In the case of underground tanks founded at h_0 = 1 m (Fig. 5) three envelopes for the tanks of the volumes V_0 = 100, 200, and 300 m^3 have been introduced, due to bigger differences in wall thicknesses. For underground tanks founded at h_0 = 10 m (Fig. 6) only two volumes V_0 = 100 m^3 (the bottom curves) and V_0 = 300 m^3 (the upper curves) have been introduced. The difference of critical wall thicknesses for these tanks is less than 3%.

Table 1
Values of parameters β_1, β_2 of the envelope (10) for ground tanks

	$p_0 = 0.01$ MPa		$p_0 = 0.1$ MPa
	$V_0 = 100\ m^3$	$V_0 = 300\ m^3$	$V_0 = 300\ m^3$
β_1	0.001094	0.001142	0.002930
β_2	0.3676	0.3640	0.4010

Table 2
Values of parameters β_1, β_2 of the envelope (10) for underground tanks

	$h_0 = 1$ m			$h_0 = 10$ m	
	$V_0 = 100\ m^3$	$V_0 = 200\ m^3$	$V_0 = 300\ m^3$	$V_0 = 100\ m^3$	$V_0 = 300\ m^3$
β_1	0,002613	0,002821	0,003080	0,003608	0,003710
β_2	0,2775	0,2715	0,2539	0,3610	0,3606

The expression (10) may be of practical meaning for designing of horizontal cylindrical tanks. Additional load of the tank ($p_0 = 0.1$ MPa or $h_0 = 10$ m) corresponds to overload of the structure. The envelopes are shown in Fig. 7 and/LINK Fig. 8 for ground and underground tanks of the volume $V_0 = 300\ m^3$.

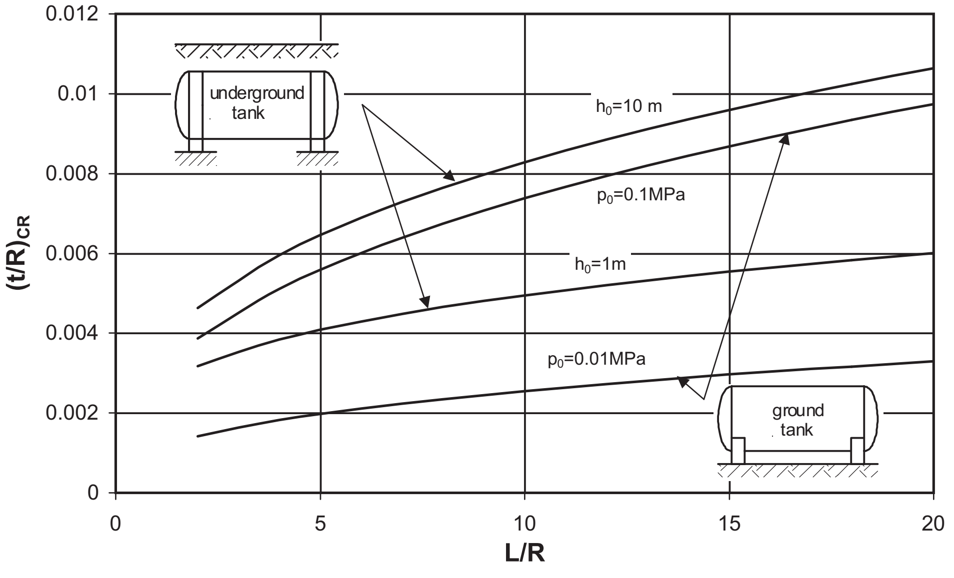

Fig. 7. Critical sizes of horizontal tanks ($V_0 = 100\ m^3$).

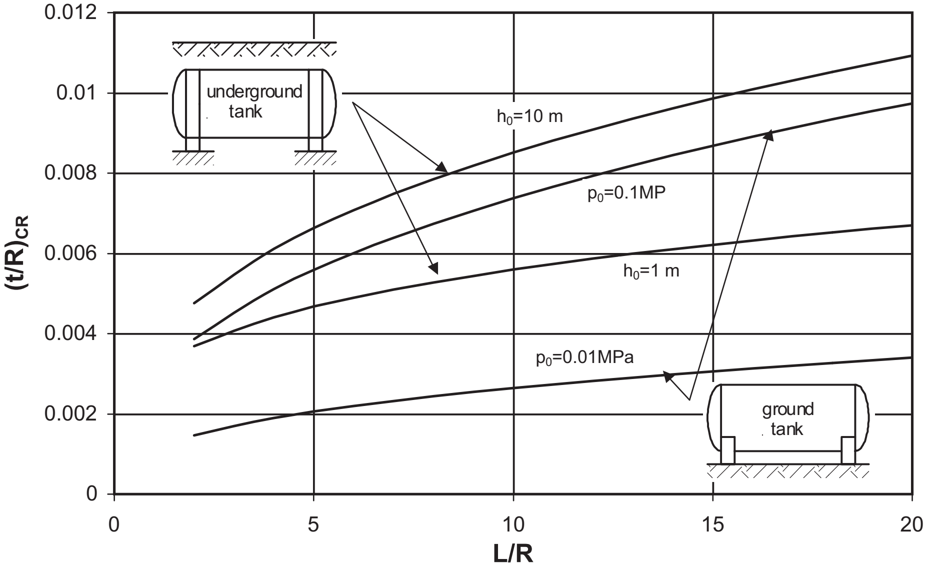

Fig. 8. Critical sizes of horizontal tanks ($V_0 = 300\ m^3$).

5. Conclusions

The wall of cylindrical shell of a ground or underground tank is subject to non-homogeneous stress. As there are the regions where compression occurs, buckling becomes possible. Equations of stability have been solved for cylindrical shell, providing critical dimensions of the structure. It was ascertained, based on numerical analysis, that in case of similar loads critical thickness of the wall of underground tank exceeds that of the ground one.

References

[1] Huang D, Redekop D, Xu B. Instability of a cylindrical shell under three-point bending. Thin-Walled Structures 1996;26(2):105–22.
[2] Guarracino F, Fraldi M. On the analysis of cylindrical tubes under flexure. Proceedings of the Third Int. Conference on Thin-Walled Structures. Elsevier, 2001:653–660.
[3] Chan CMG, Tooth AS, Spence J. A study of the buckling behaviour of horizontal saddle supported vessels. Thin-Walled Structures, 1998;30(1–4):3–22.
[4] Tooth AS, Spence J, Chan GCM. The buckling behaviour of horizontal storage vessels — Experiments and codes. In: Shanmugam NE, editor. Thin-Walled Structures, Research and Development. 1998. p. 597–604.
[5] Kacperski T. Experimental study of the phenomenon of stability loss of a horizontal thin-walled cylindrical shell loaded by a liquid. Eng. Transactions 1994;42(4):331–7.
[6] Saal H. Buckling of long liquid-fill cylindrical shells. In: Ramm, editor. Buckling of Shells. Berlin, Heidelberg, New York: Springer-Verlag; 1982. p. 333–52.

[7] Magnucki K, Stasiewicz P. Theoretical analysis of buckling of liquid-filled cylindrical shell. VIIIth Symposium on Stability of Structures, Zakopane 1997:163-168 (in Polish).
[8] Magnucki K, Stasiewicz P. Critical state of circular cylindrical horizontal tank filled with liquid. Xth International Scientific-Technical Conference ‘Metal Structures’, Gdańsk 2001;3:153–160 (in Polish).
[9] Stasiewicz P. Critical thickness of the circular shell loaded by hydrostatic and uniform external pressure. Pedagogical University, Study and Materials, XLVIII, 1, Zielona Góra 1999;141–149 (in Polish).
[10] Magnucki K. Optimal design of cylindrical vessels under strength and stability constraints, Proceedings 9th Int. Conference on Pressure Vessel Technology, Sydney 2000;1:867–874.
[11] Ziółko J. Metal tanks for liquid and gas. Arkady, Warsaw 1986 (in Polish).
[12] Tovstik PE. Stability of thin-walled shells. Nauka-Fizmatlit, Moscow 1995 (in Russian).